Coupling Conduction, Convection and Radiative Transfer in a Single Path-Space: Application to Infrared Rendering

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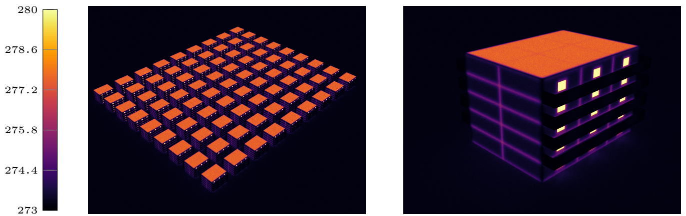
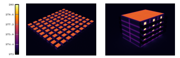
Solving heat-transfer for infrared rendering



Conduction Convection Radiation

Solving heat-transfer for infrared rendering

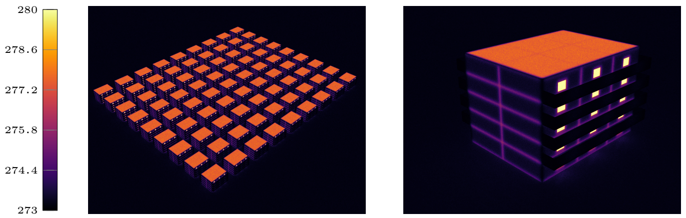


Conduction Convection Radiation

We want to solve heat-transfer in complex scenes, like this infrared rendering of a city.
The question is challenging because we need to solve three different physics: conduction, convection and radiative transfer which are coupled.
And we need to do this at very different scales: from centimeters, like for heat conduction through the glass of windows, up to tens of meters for radiation between buildings.

So far, heat transfer physicists have addressed this question in two ways.

Solving heat-transfer for infrared rendering

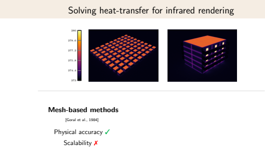


Mesh-based methods

[Goral et al., 1984]

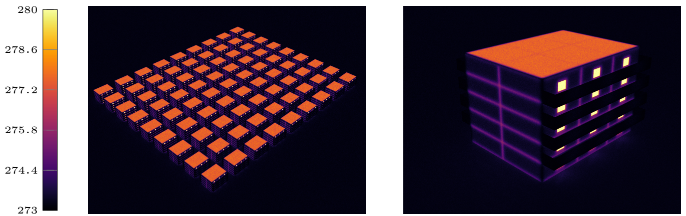
Physical accuracy ✓

Scalability ✗



The first way is to first solve the temperature field over the entire scene, using a deterministic method ; then make an infrared image of this field as we would do with radiosity.
However, deterministic methods require a detailed volumetric mesh on which to solve the coupled physics.
This does not scale up to a city... well, actually, it can even be complicated for a single building!

Solving heat-transfer for infrared rendering



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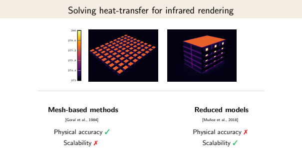
Reduced models

[Muñoz et al., 2018]

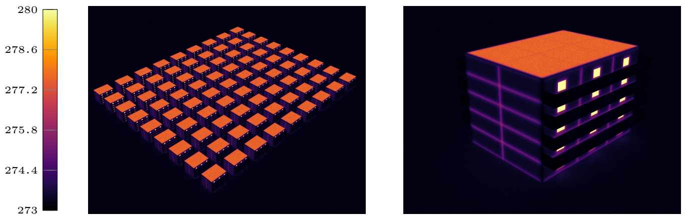
Physical accuracy ✗

Scalability ✓



Another way is to degrade the physics by using reduced models, like equivalent electrical circuits. However, these reduced models cannot guarantee the physical accuracy of the solution.

Solving heat-transfer for infrared rendering



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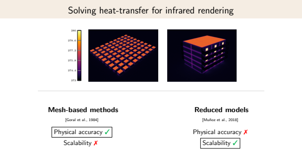
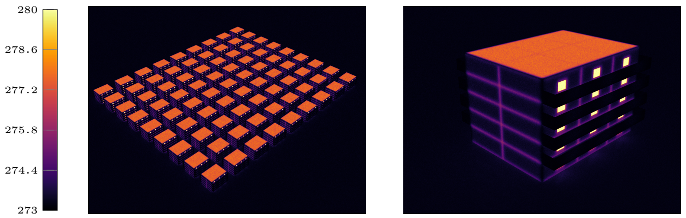


Here, I will present a way to ensure both physical accuracy *and* scalability.

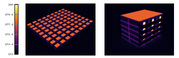
Solving heat-transfer for infrared rendering



1 space

Conduction	Convection	Radiation

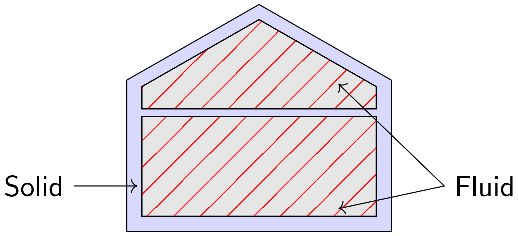
Solving heat-transfer for infrared rendering

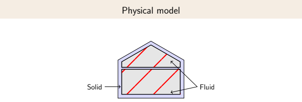


1 space
Conduction Convection Radiation

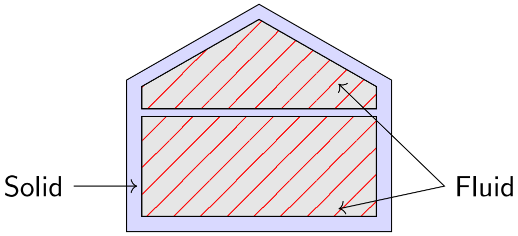
For doing so, our proposal is to reformulate the physics, namely conductive, convective and radiative transfers into a *unique* space of *coupled* paths.
Once this unique path-space has been constructed, we sample it by Monte Carlo method, and we directly benefit from the many advantages of that method, including scalability.

Physical model





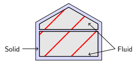
Our system is composed of fluids and solids.
For didactic reasons, we choose simple physical models.
Extensions are discussed in the paper.



Conduction

Diffusion

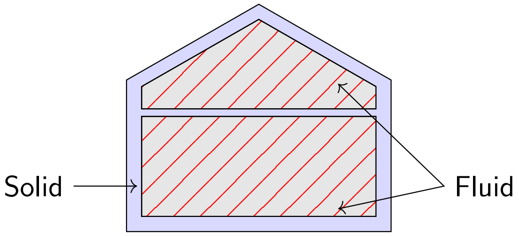
Physical model



Conduction
Diffusion

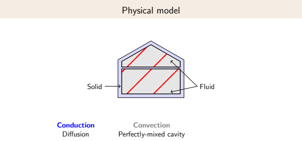
In solids, conduction is modeled by a diffusion equation on the temperature.

Physical model



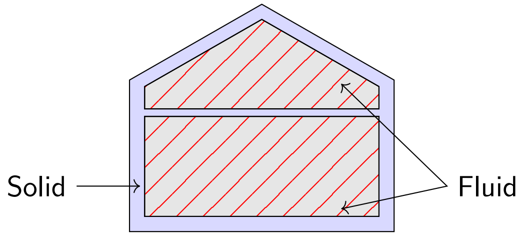
Conduction
Diffusion

Convection
Perfectly-mixed cavity



For convection in fluid cavities, we assume a model of a perfectly-mixed cavity, so the temperature is homogeneous within a given volume but may vary over time.

Physical model

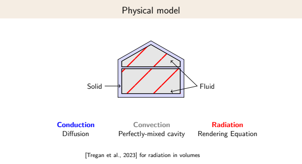


Conduction
Diffusion

Convection
Perfectly-mixed cavity

Radiation
Rendering Equation

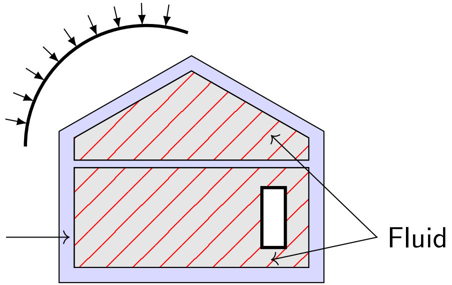
[Tregan et al., 2023] for radiation in volumes



For radiation, we only consider here radiative transfer between the fluid cavity walls, modeled by the Rendering Equation on radiance.

Of course, radiative transfer could also appear in volumes as described by Tregan, replacing the Rendering Equation by the Radiative Transfer Equation, but we do not consider this case here.

Physical model



Conduction
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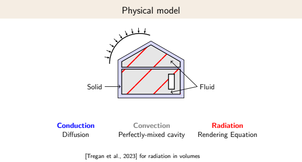
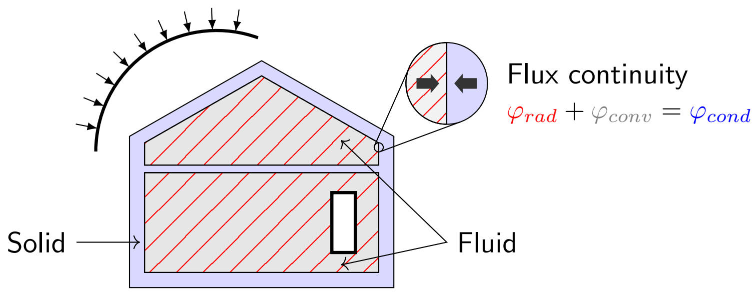


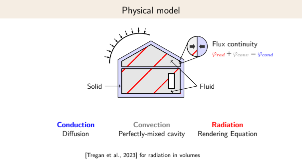
At the boundaries of the system, the temperature is prescribed, here for example the temperature of the radiative environment and a heater device.

Physical model



Conduction	**Convection**	**Radiation**
Diffusion	Perfectly-mixed cavity	Rendering Equation

[Tregan et al., 2023] for radiation in volumes



In such a system, coupling appears only at interfaces.
This is where transitions from one transfer mode to another are considered.
They are treated based on the flux continuity.

Now, ... How do we build a *single* space of paths coupling conduction, convection and radiation?

There are 4 key steps.

Coupled path-space construction

1. Express all equations on the same quantity

Rendering Equation

$$L = \epsilon L^{eq}(\theta) + (1-\epsilon) \int_{2\pi} L_i \; p_r(\omega|\omega_i) \; d\omega_i \quad \text{with} \quad L = \frac{\sigma \theta_R^4}{\pi} \approx \frac{\sigma \theta_{ref}^3}{\pi} (\theta_R - \theta_{ref})$$

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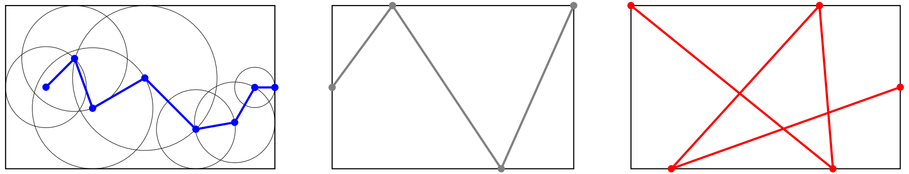
First, we express all equations with respect to the same quantity: temperature.
So here, we write the Rendering Equation in terms of temperature instead of radiance, and we linearize its expression.

Coupled path-space construction

1. Express all equations on the same quantity
2. For each mode, write the quantity as an expectation of a process



Conduction   Convection   Radiation

Coupled path-space construction

1. Express all equations on the same quantity
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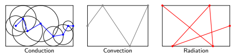
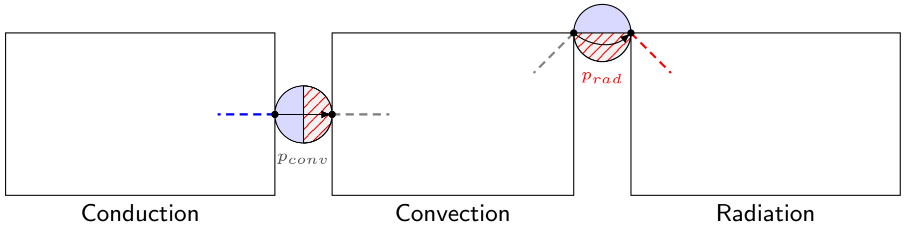


Second, for each mode of transfer taken separately, temperature is written as the expectation of a process.
Using Feynman-Kac work, we can then construct a path-space for each mode.
Here you see examples of paths for conduction, convection and radiation.
Now, in order to couple these modes...

Coupled path-space construction

1. Express all equations on the same quantity
2. For each mode, write the quantity as an expectation of a process
3. Probabilize the coupling between modes



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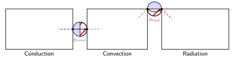
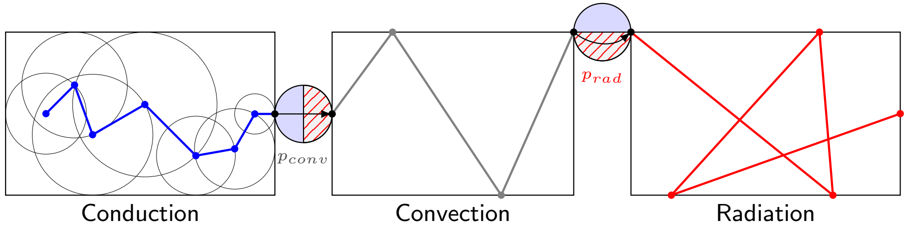


Third, we also probabilize the coupling which occurs at interfaces.
For this, we use the flux continuity equation at an interface to write the temperature as an expectation depending on the modes of transfer occurring at both sides of it.

Coupled path-space construction

1. Express all equations on the same quantity
2. For each mode, write the quantity as an expectation of a process
3. Probabilize the coupling between modes
4. Apply Monte Carlo to build a single path-space



Coupled path-space construction

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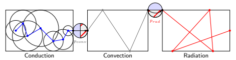
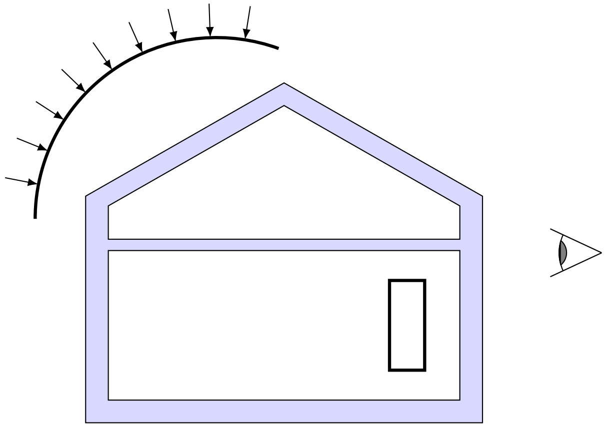
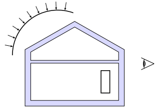


At this stage, the three modes of transfer, as well as their coupling by interfaces, are all formally described by their physically-exact expectation.
Hence, fourth, using the *double randomization* principle, we can write a Monte Carlo algorithm to sample a *single* space of paths.
Starting from a probe point, one path can now be sampled back to a source, by switching between transfer modes at interfaces.

Monte Carlo algorithm

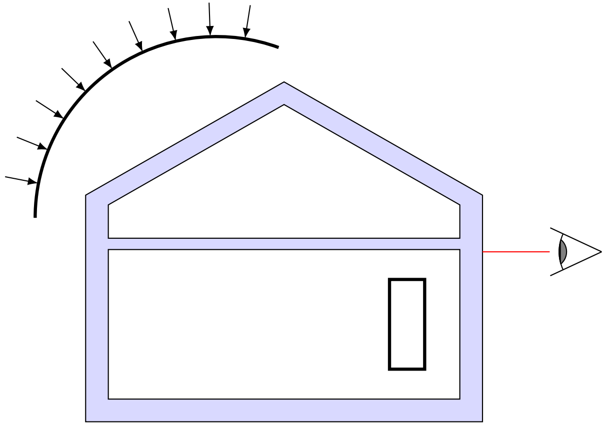


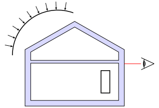


Here is an illustration of the algorithm.

The path starts from a position, at a time and in a physical mode which depend on the sensor.

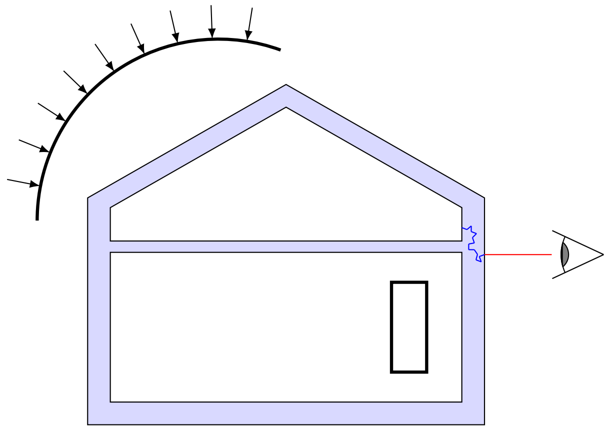
Monte Carlo algorithm

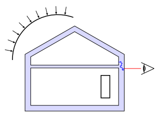




For an infrared rendering, the path starts from the camera in radiative mode.
It propagates until it encounters an interface.
If the surface temperature is unknown,

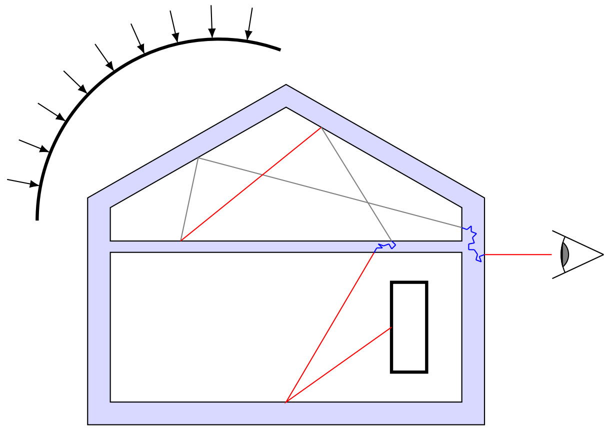
Monte Carlo algorithm

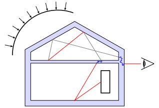




then the next mode of transfer is sampled by importance, depending on the physics.
The path propagates in this mode until it reaches the next interface, ...

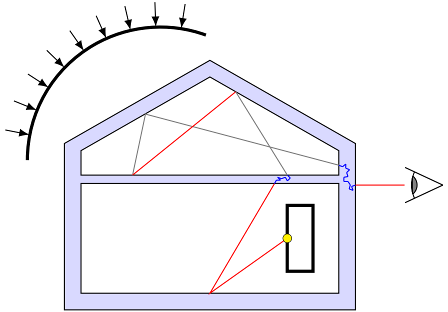
Monte Carlo algorithm



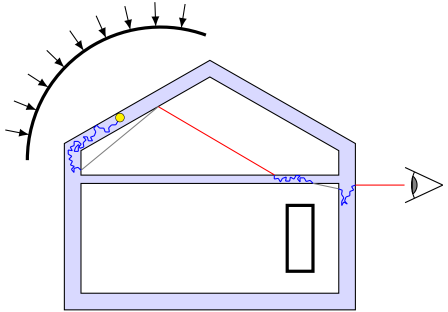


where a next mode is sampled, and so on.
Note that conduction and convection are inertial phenomena, so we progress backward in time along propagation.

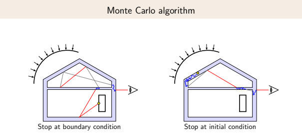
Monte Carlo algorithm



Stop at boundary condition



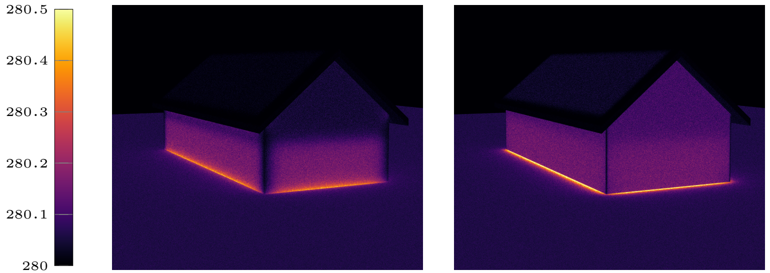
Stop at initial condition

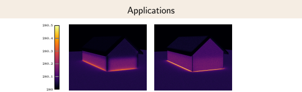


The path stops when it finds a boundary condition or the initial condition.
In the first case, the prescribed temperature is taken as the weight of the realization.
In the second case, the weight is given by the initial condition.

The value at the sensor is estimated by averaging these weights over many realizations.

Applications

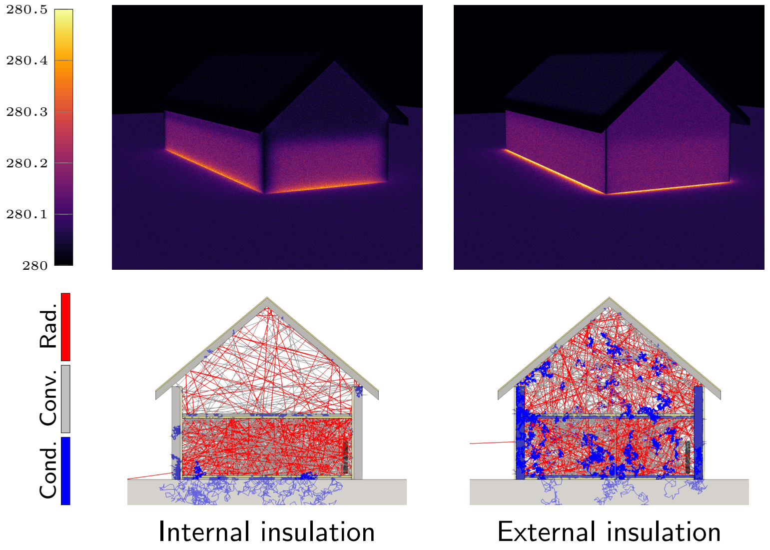


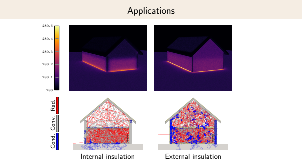


I will now present some applications of this algorithm to infrared rendering.
Here we have a farm with a heater in the lower room, with two different insulation: either from the inside or the outside...

Could you guess which image corresponds to which insulation?

Applications

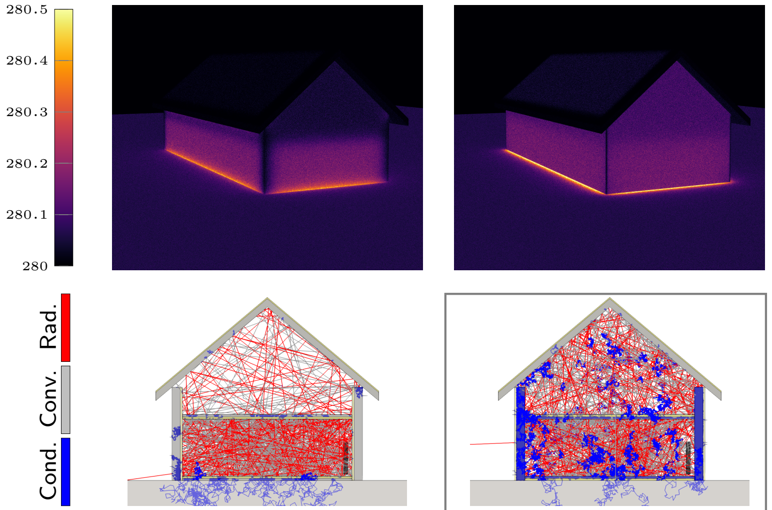




Well, if you are not a thermal engineer, like me, studying the paths distribution can help you to understand what makes the difference.

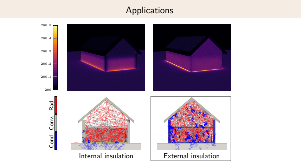
Here they start from the lower room.

Applications



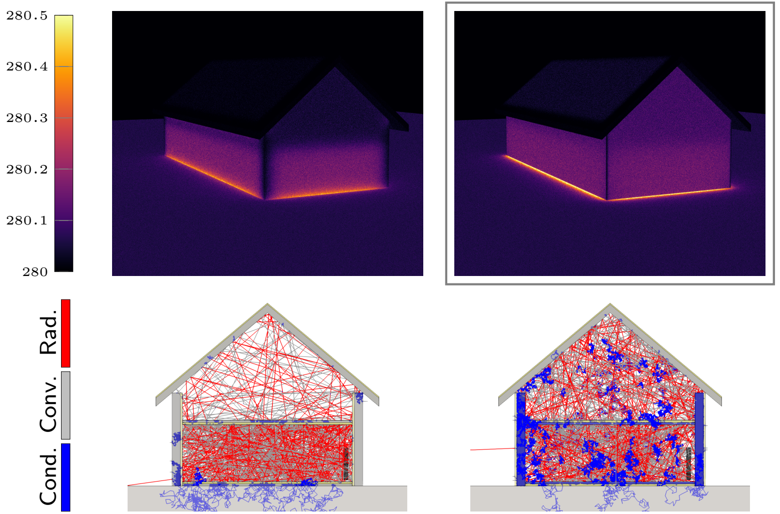
Internal insulation

External insulation

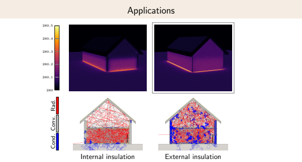


When the insulation is from the outside, you can see that a lot of paths propagate through the walls and reach the upper room.

Applications

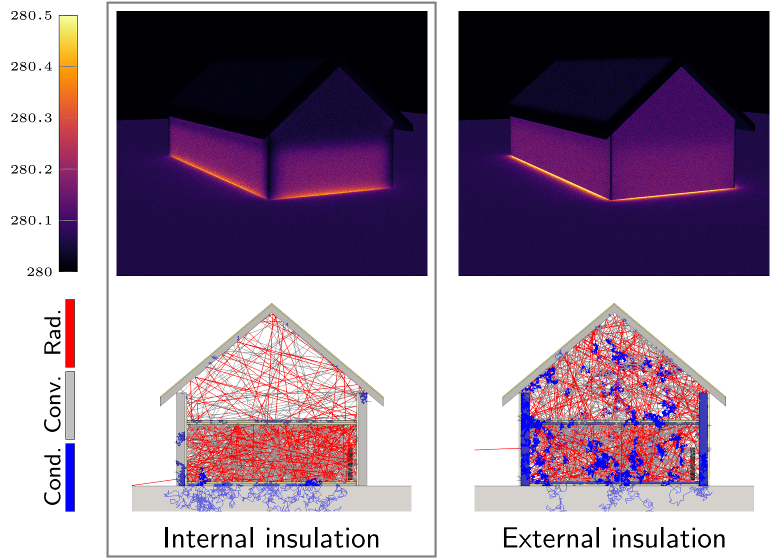


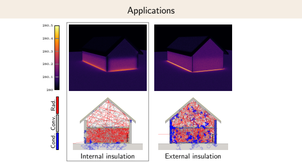
Internal insulation External insulation



This results in a more homogeneous temperature across the gabble.

Applications





On the contrary, when the insulation is from the inside, paths tend to stay in the lower room. So, a first point is that visualizing the paths can greatly help the analysis of thermal behavior of a building, or a city.

Applications

- Path visualization for analysis
- Fast model through path replay

Change source temperature : store final source identifier

Applications

- Path visualization for analysis
- Fast model through path replay

Change source temperature : store final source identifier

Second, we can store the identifier of the source at the end of paths, instead of boundary temperatures as Monte Carlo weights.
Since our physics is linear, we then build a *propagator*, and this propagator can be used as a fast model to replay paths with different initial and boundary conditions, at no additional cost.

Applications

- Path visualization for analysis
- Fast model through path replay

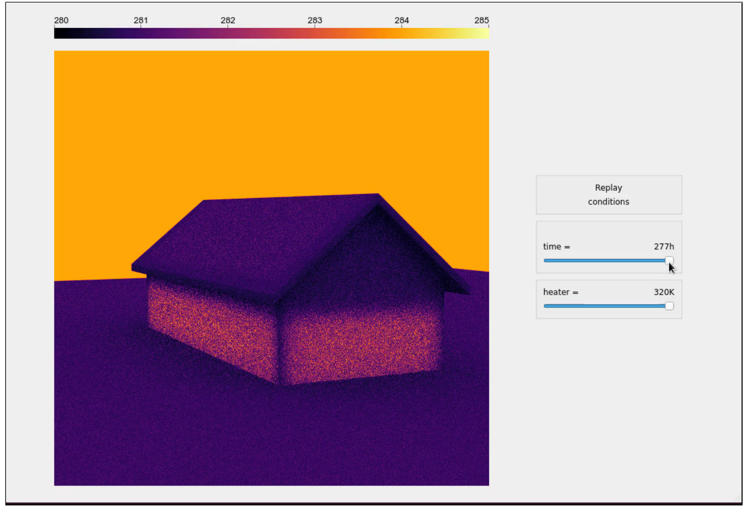
Change source temperature : store final source identifier
or observation time + path duration*

*Under homogeneous initial condition

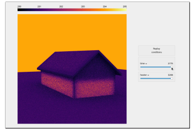
Applications
Path visualization for analysis
Fast model through path replay
Change source temperature : store final source identifier
or observation time + path duration*
*Under homogeneous initial condition

In addition, if we want to change the observation time, we can also store the path duration. This information is sufficient when the initial temperature is homogeneous.

Applications



Applications



Click on the image to play the video.
Having computed the paths once for all, we replay them almost instantaneously.
By changing the observation time, we obtain a propagation film of the transient state.
Alternatively, we can change the temperature of a source, like the heater device.
This fast model is useful for many applications, such as control, or for optimization and inversion.

Note again that the fast model still preserve the physical accuracy!

Applications

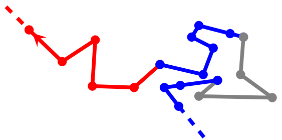
- Path visualization for analysis
- Fast model through path replay
- Parameter sensibility analysis [Penazzi et al., 2022]

Applications

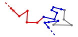
- Path visualization for analysis
- Fast model through path replay
- Parameter sensibility analysis [Penazzi et al., 2022]

Finally, the use of Monte Carlo also opens up the possibility of sensitivity studies, not discussed in the present paper, but presented by Penazzi in the same context of heat-transfer.

Perspectives – linear situations



Coupling via the sources

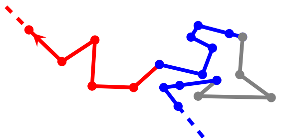


Coupling via the sources

So, we have presented a methodology to solve heat transfer for certain kind of situations, which we call "coupling via the sources" in the paper.

It results in paths alternating between modes of transfer.

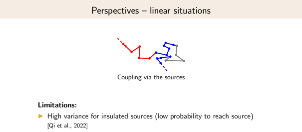
But, even if the problem is linear, some limitations remain.



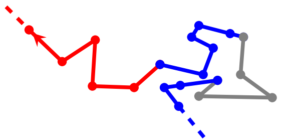
Coupling via the sources

Limitations:

- High variance for insulated sources (low probability to reach source) [Qi et al., 2022]



First, when the source is well isolated, we can end up with high variances, as it is classical in rendering.
Computer Graphics techniques such as bidirectional approaches should help in reducing this variance.

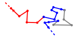


Coupling via the sources

Limitations:

- High variance for insulated sources (low probability to reach source) [Qi et al., 2022]
- Path entrapment (low probability to change mode)

Perspectives – linear situations



Coupling via the sources

Limitations:
High variance for insulated sources (low probability to reach source) [Qi et al., 2022]
Path entrapment (low probability to change mode)

Second, path can remain blocked for a very long time in one part of the system, and visit it again and again because of the set of probabilities.
Such path-entrapment greatly increases the computation time for each path, without impacting variance.

These two challenges arise in the context of a *linear* coupling via the sources.

Perspectives – towards non-linear situations

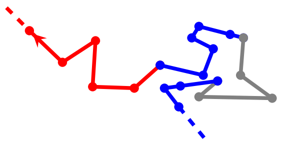
"Monte Carlo methods are not generally effective for nonlinear problems mainly because expectations are linear in character."
[Curtiss, 1953]

Perspectives – towards non-linear situations

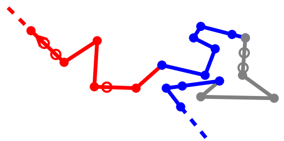
"Monte Carlo methods are not generally effective for nonlinear problems mainly because expectations are linear in character."
[Curtiss, 1953]

Now, we wonder if we can solve broader coupled physics in a single path-space?
Well, many problems are coupled in a *non-linear* way.
So a priori, Monte Carlo does not apply...
But in fact, there are at least two counter-examples where Monte Carlo is used to solve *non-linear* problems.

Perspectives – towards non-linear situations



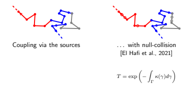
Coupling via the sources



. . . with null-collision
[El Hafi et al., 2021]

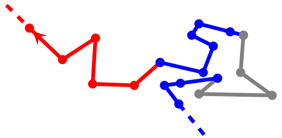
$$T = \exp\left(-\int_{\Gamma} \kappa(\gamma) d\gamma\right)$$

Perspectives – towards non-linear situations

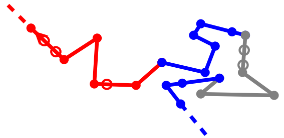


The first one involves null collisions, which is a classical trick used to deal with heterogeneous media.
For example, the integration in an exponential term of a heterogeneous extinction coefficient along a line of sight can be done using a majorant of this field.
El Hafi claims that null collisions can be considered as a way to solve *this kind of* non-linearity.

Perspectives – towards non-linear situations



Coupling via the sources



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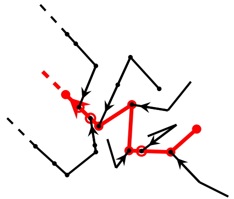
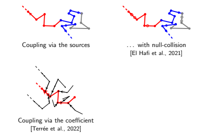


Coupling via the coefficient
[Terrée et al., 2022]

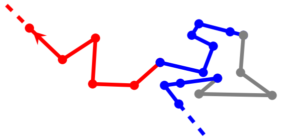
Perspectives – towards non-linear situations



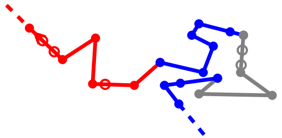
Now, what if the coefficient is no longer prescribed but is given by another *model*?
Again, we can apply the null collisions technique, as proposed by Terree.
The coupling of the models changes the structure of the path-space, but it does not lead to a combinatory explosion.

However, it requires a majorant of the coefficient... and it is not always possible to define it!

Perspectives – towards non-linear situations



Coupling via the sources



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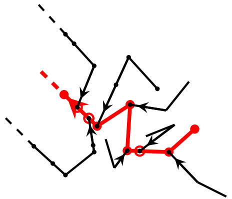

Coupling via the coefficient
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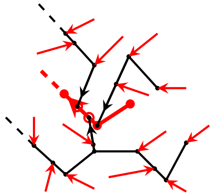
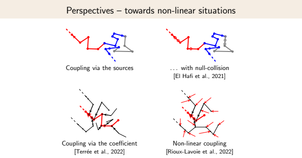
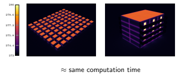


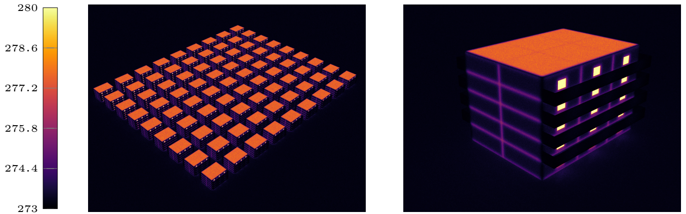
Non-linear coupling
[Rioux-Lavoie et al., 2022]



Finally, there are situations when the coefficient also depends on the quantity to be solved itself, here temperature.
In such cases, the path-space is completely branched, as in Rioux-Lavoie's graphics paper for fluids.

These potential extensions represent first steps towards the handling of non-linearities with Monte-Carlo.
Of course, many challenges remain, in particular, the long-term prospect of solving the Navier-Stokes equations by Monte Carlo.





≈ same computation time

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