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Radiative, conductive and convective heat-transfers in a single Monte Carlo algorithm

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Abstract. It was recently shown that null-collision algorithms could lead to grid-free radiative-transfer Monte Carlo algorithms that immediately benefit of computer-graphics tools for an efficient handling of complex geometries [1, 2]. We here explore the idea of extending the approach to heat transfer problems combining radiation, conduction and convection. This is possible as soon as the model can be given the form of a second-kind Fredholm equation. In the following pages, we show that this is quite straightforward at the stationnary limit in the linear case. The oral presentation will provide corresponding simulation examples. Perspectives will then be drawn concerning the extension to non-stationnary cases and non-linear coupling.

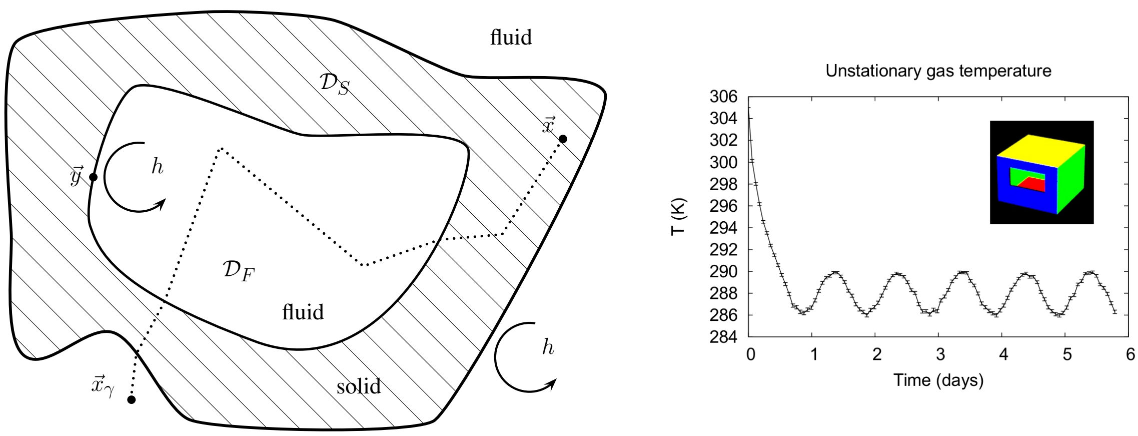

Figure 1. Fluid and solid domains (left). Academic example of a Monte Carlo simulation dealing with radiative, conductive and convective heat-transfers, for opaque solids and transparent fluids (internal air-tempertaure for a sinusoidal external forcing, right).

We consider a system constituted of solid and fluid connex domains, noted $\mathcal{D}_S$ and $\mathcal{D}_F$ respectively, of boundary $\partial\mathcal{D}_S$ and $\partial\mathcal{D}_F$. The heat transfer modes are conduction/radiation in the solid, and convection/radiation in the fluid. Both the solid and the fluid are semi-transparent and grey. Density ρ, specific heat capacity C, conductivity λ and absorption coefficient k_a in the solid are all heterogeneous but independant of temperature. The fluid is assumed perfectly mixed in each connex domain : ρ, C and k_a are therefore homogeneous. They are also independent of temperature. Convective heat transfer is resumed to an heat exchange coefficient h at each location at the solid boundary, and an incoming/outcoming flow of volumetric flow-rate ϕ, the incoming flow being of a known temperature θ_N. These three quantities are independent of the solid and fluid temperatures. Radiation is linearised around a reference temperature θ_{ref}, defining a volumetric radiative heat transfer coefficient $\zeta = 16 k_a \sigma \theta_{ref}^3$, where σ is the Stefan-Boltzmann constant. Without precising the initial conditions, the resulting heat-transfer model is linear in temperature:

$$\left.\begin{aligned} \vec{x} \in \mathcal{D}_S \ :& \quad \rho C \frac{\partial \theta}{\partial t} = -\vec{\nabla}.\left(-\lambda \vec{\nabla}\theta\right) + \zeta(\theta_R - \theta) \\ \vec{x} \equiv \vec{y} \in \partial\mathcal{D}_S \ :& \quad -\lambda \vec{\nabla}\theta.\vec{n} = h(\theta_F - \theta) \end{aligned}\right\} \quad \text{solid} \tag{1}$$

$$\left.\begin{aligned} \rho C \mathcal{V}_F \frac{\partial \theta}{\partial t} = \rho C \phi(\theta_N - \theta) \ &+ \ \zeta \int_{\mathcal{D}_F} (\theta_R(\vec{x}_R, t) - \theta) d\vec{x}_R \\ &+ \ \int_{\partial\mathcal{D}_S} h(\vec{y}, t)(\theta_S(\vec{y}, t) - \theta) d\vec{y} \end{aligned}\right\} \quad \text{fluid} \tag{2}$$

$$\left.\theta_R = \int_{\mathcal{D}_\Gamma} p_\Gamma(\gamma) d\gamma \ \theta(\vec{x}_\gamma, t - t_\gamma)\right\} \quad \text{solid or fluid} \tag{3}$$

where θ is temperature, t is time, $\vec{x}$, $\vec{x}_R$ (and later $\vec{x}_G$) are locations within the solid or fluid domains, and $\vec{y}$ (and later $\vec{y}_G$) is a location at the boundary. At a boundary location, θ_S and θ_F are respectively the solid and fluid temperatures; $\vec{n}$ is the unit normal heading toward the solid. The radiative temperature θ_R is defined using the space $\mathcal{D}_\Gamma$ of all trajectories γ of photons emitted at $\vec{x}$, until their absorption at $\vec{x}_\gamma$. The probability density $p_\Gamma(\gamma)$ of trajectory γ reflects the statistical pictures associated to the radiative transfer equation and θ_R is therefore simply the average value of the temperature at absorption locations, for photons emitted at $\vec{x}$.

As the problem is linear, its general solution can be expressed using an integral formulation involving unstationnary Green functions. We here only give the stationnary solution:

$$\left.\begin{aligned} \vec{x} \in \mathcal{D}_S \ : \quad \theta(\vec{x}) = &\int_{\mathcal{D}_S} d\vec{x}_G \ G_{S,stat}(\vec{x};\vec{x}_G) \ \theta_R(\vec{x}_G) \\ &+ \int_{\partial\mathcal{D}_S} d\vec{y}_G \ G_{S,stat}(\vec{x};\vec{y}_G) \frac{h(\vec{y}_G)}{\zeta(\vec{y}_G)} \ \theta_F(\vec{y}_G) \end{aligned}\right\} \quad \text{solid} \tag{4}$$

$$\left.\theta = \frac{\rho C \phi \theta_N \ + \ \zeta \int_{\mathcal{D}_F} \theta_R(\vec{x}_R) d\vec{x}_R \ + \ \int_{\partial\mathcal{D}_S} h(\vec{y}) \theta_S(\vec{y}) d\vec{y}}{\rho C \phi \ + \ \zeta \mathcal{V}_F \ + \ \int_{\partial\mathcal{D}_S} h(\vec{y}) d\vec{y}}\right\} \quad \text{fluid} \tag{5}$$

$$\left.\theta_R = \int_{\mathcal{D}_\Gamma} p_\Gamma(\gamma) d\gamma \ \theta(\vec{x}_\gamma)\right\} \quad \text{solid or fluid} \tag{6}$$

where $\mathcal{V}_F$ is the volume of $\mathcal{D}_F$, and $G_{S,stat}(\vec{x};\vec{x}_G)$ is the value of the stationnary Green function, at $\vec{x}$ within the solid, for a source at $\vec{x}_G$ within the same solid domain.

These integrals can each be interpreted as expectations of random variables and their combination allows to interprete the temperature at any location, in the solid or in the fluid, as the expectation of a random function $\Theta(\vec{x})$. The Monte Carlo algorithm then evaluates $\theta(\vec{x})$ as the average of a large number of Θ samples [3]. The definition of Θ combines six random functions of location: A_N, A_R and $A_N C$ for the selection a source type (incoming flow, radiation or convection), $\vec{X}_G$ for the sampling of a location within the solid or fluid domain, $\vec{Y}_G$ for sampling of a location at the boundary and Θ_R that samples of an optical path and returns the temperature at its extremity.

$$\left.\begin{aligned} \theta(\vec{x}) &= \mathscr{E}\left[\Theta(\vec{x})\right] \\ \Theta(\vec{x}) &= A_N(\vec{x})\theta_N + A_R(\vec{x})\Theta_R(\vec{X}_G(\vec{x})) + A_C(\vec{x})\theta_C(\vec{Y}_G(\vec{x})) \end{aligned}\right\} \quad \text{solid or fluid} \tag{7}$$

As $\Theta_R(\vec{X}_G(\vec{x}))$ and $\theta_C(\vec{Y}_G(\vec{x}))$ involve the solid and fluid temperatures at random locations, these temperatures can again be replaced by the expectation of Θ at the corresponding locations and the Monte Carlo algorithm becomes recursive, just as for simulation of multiply scattered radiation. The algorithm stops when the required temperature is known, either because this corresponds to the incoming flow and θ_N is known, or because the sampled location is at the boundary. This recursivity is a typical consequence of the combination of equations 4, 5 and 6 leading to a second-kind Fredholm equation.

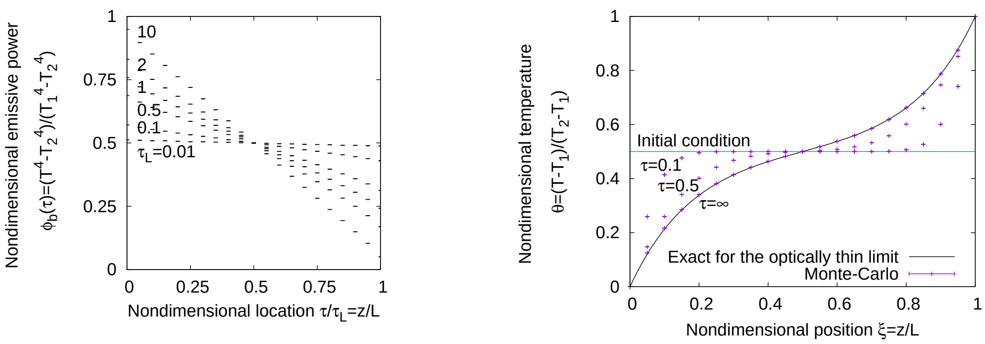

Figure 2. Academic 1D examples involving semi-transparent solids: radiative "equilibrium" (left); radiation coupled with non-stationnary conduction (right). Notations are from [4] page 460 and page ??.

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